Exercise 97

Use the Chain Rule to show that if θ is measured in degrees, then

$$\frac{d}{d\theta}(\sin\theta) = \frac{\pi}{180}\cos\theta$$

(This gives one reason for the convention that radian measure is always used when dealing with trigonometric functions in calculus: the differentiation formulas would not be as simple if we used degree measure.)

Solution

The argument of sine has to be a number here, not an angle; if θ is in degrees, it must be converted to radians before it can be manipulated with the tools of calculus. Differentiate the sine function using the chain rule.

$$\begin{aligned}
\frac{d}{d\theta}(\sin\theta) &= \frac{d}{d\theta}\left(\sin\frac{\pi\theta}{180}\right) \\
&= \left(\cos\frac{\pi\theta}{180}\right)\cdot\frac{d}{d\theta}\left(\frac{\pi\theta}{180}\right) \\
&= \left(\cos\frac{\pi\theta}{180}\right)\cdot\left(\frac{\pi}{180}\right) \\
&= \frac{\pi}{180}\cos\frac{\pi\theta}{180} \\
&= \frac{\pi}{180}\cos\theta
\end{aligned}$$